Announcements

- Short hw3 due today.

  Small clarification on problem 1c pinned on Ed

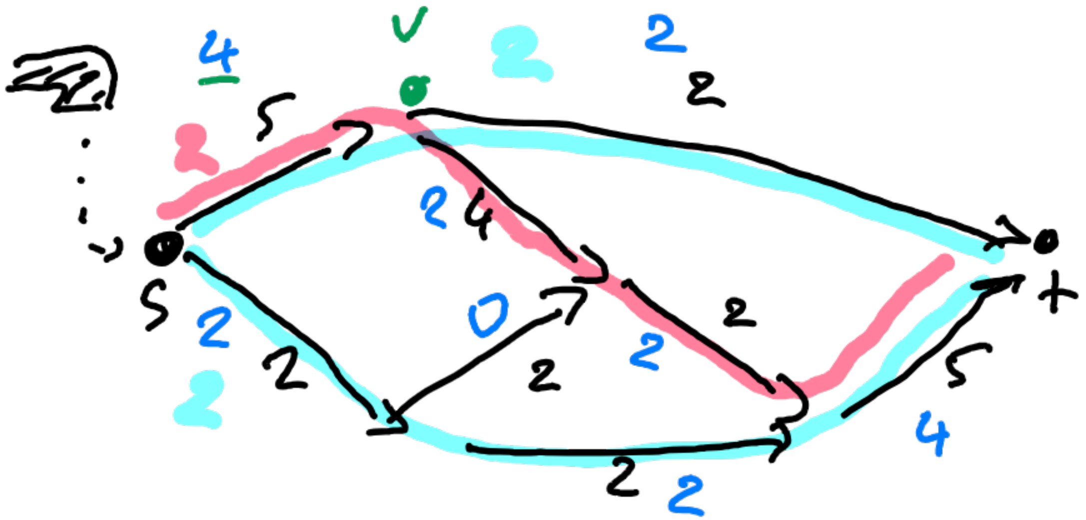
- Poll Everywhere credit is posted on Canvas, starting Feb 2

- Hw4 will be out after class (1 coding, 2 written questions)

- Updated course policies on the Web to include all policies (dropped quiz, regrade deadline, etc)

- Monday & Tuesday section: practice with flows and stable matching, no quiz

The maximum flow problem



Input network = directed graph

$G = (V, E)$

$s, t \in V$ source & sink

$c_e \geq 0 \quad e \in E$ capacities

Assume no (v, s) edges in network

Problem: find flow of maximum value

flow $\quad 0 \leq f_e \leq c_e$ capacity constraint

$\forall v \neq s,t \quad \sum_{w:\, wv \in E} f_{wv} = \sum_{w:\, vw \in E} f_{vw}$ flow conservation

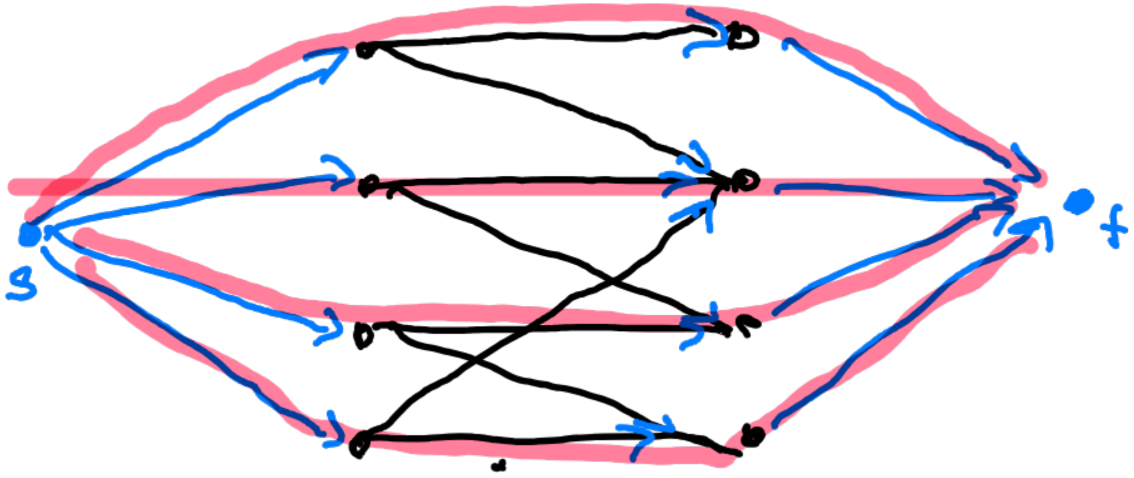
flow into(v) = flow out of(v)

$\text{value}(f) = \sum_{v:\, sv \in E} f_{sv}$

Maximum flow example: matching

reduce maximum size matching to flow

capacity $c_e = 1$ all edges



Fact (last time)

maximum integer flow

= max size matching

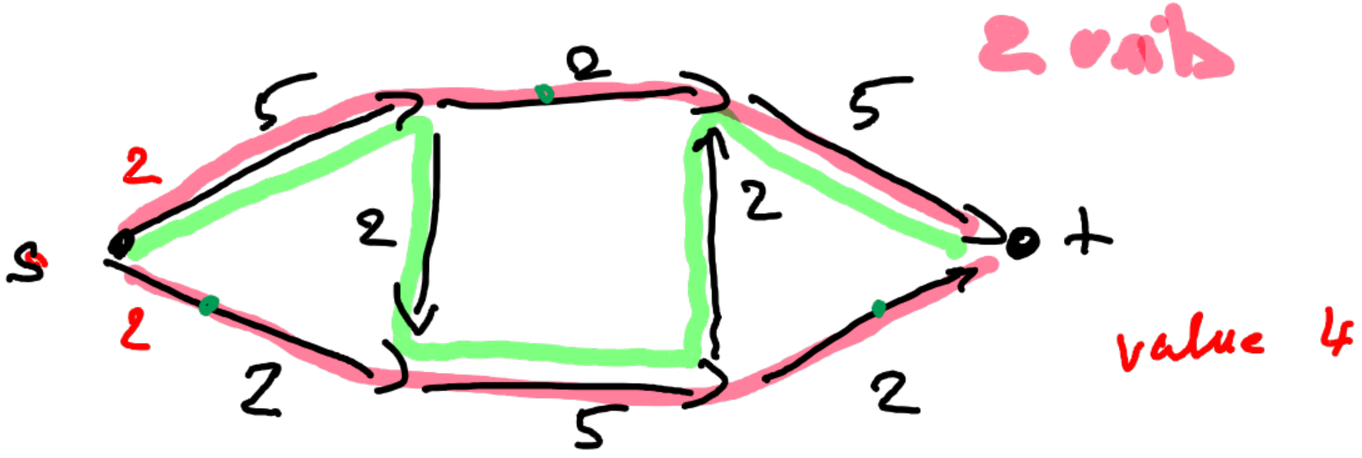
$e \in M$ if & only if $f_e = 1$

Join by Web PollEv.com/evatardos772

Is this flow of maximum value?

A. yes

B. no



there is a path s-t P such that $f_e < c_e$ on all $e \in P$

Augmenting path

Attempt 1 for an algorithm: augmenting path

set $f_e = 0 \ \forall e$

while $\exists$ augmenting path

Select edges $E^+ = \{e : f_e < c_e\}$

find $s \rightsquigarrow t$ path in E^+

$\delta = \min_{e \in P} (c_e - f_e)$

$f_e = \begin{cases} f_e + \delta & e \in P \\ f_e & \text{otherwise} \end{cases}$

endwhile

flow into(v) = flow out of(v)

Does the algorithm above find a maximum flow on this network?

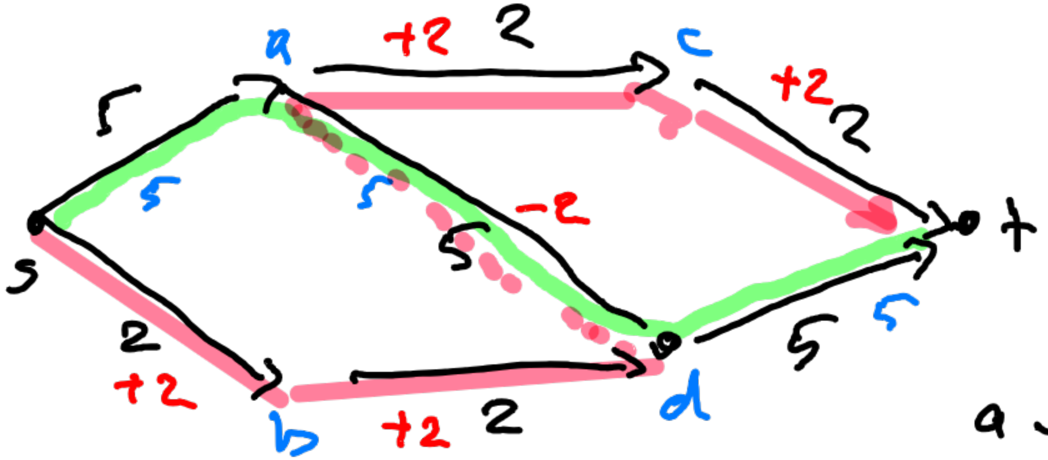
A. yes

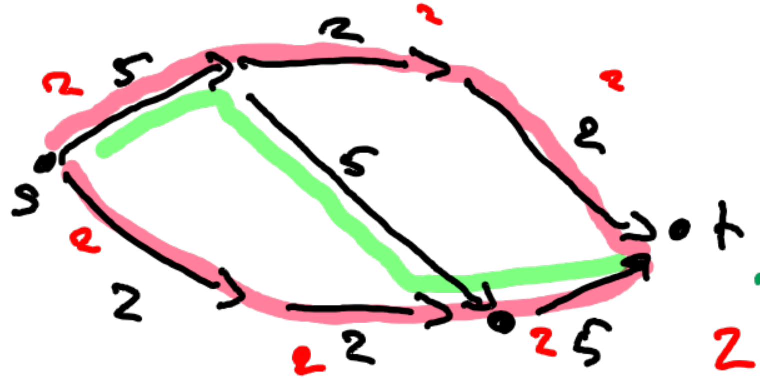
B. no

capacities

send 5 units on path s-a-d-t value = 5

send flow back



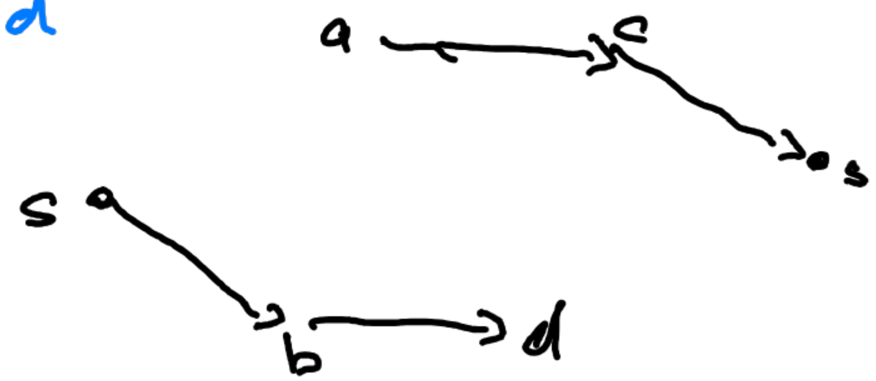


+3 on green

2 each

value 7

E^+



Ford Fulkerson algorithm

Ideas on fixing algorithm: select path in better order??

- shortest first
- min capacity first

Start $f_e = 0 \quad \forall e$

While find s→t path

$$E_f = \begin{cases} (v,w) & \text{if } e=(v,w)\in E \;\&\; f_e < c_e \\ (w,v) & \text{if } e=(v,w)\in E \;\&\; f_e > 0 \end{cases}$$

— forward, f_e can increase

— backward, f_e can decrease

find path P from s-t in E_f ← residual graph

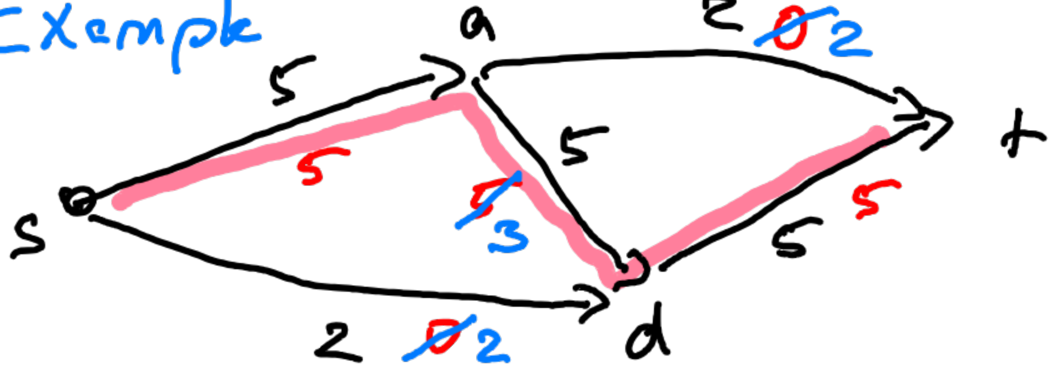
$$\delta = \min\Big(\min_{\substack{e\in P\\ \text{forward}}} (c_e - f_e),\ \min_{\substack{e\in P\\ \text{backward}}} f_e\Big)$$

$$f_e = \begin{cases} f_e+\delta & \text{if } e\in P \text{ forward} \\ f_e-\delta & \text{if } e\in P \text{ backwards} \\ f_e & \text{otherwise} \end{cases}$$

endwhile

Properties and running time

Example

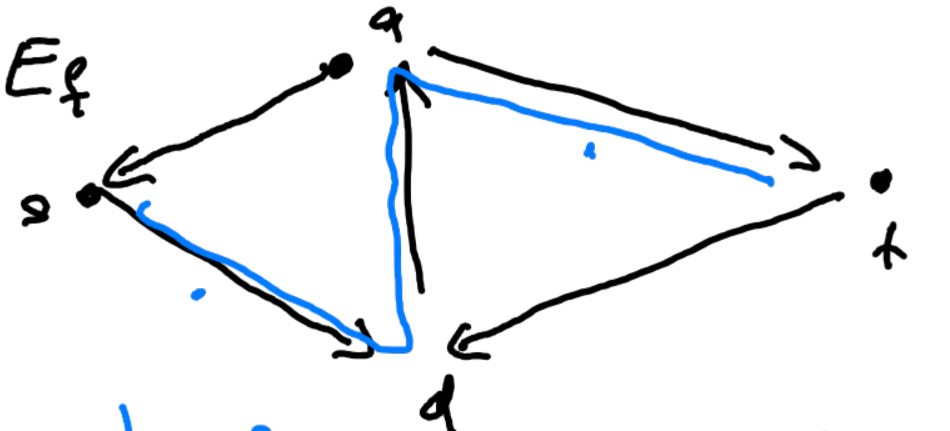


flow after step 1

update step 2

step 0: $f_e = 0$

step 1: s a d t : $\delta = 5$



step 2: blue path $\delta = 2$

Properties and running time

(1) Valid flow at every step

by induction base case $f_e = 0$ valid

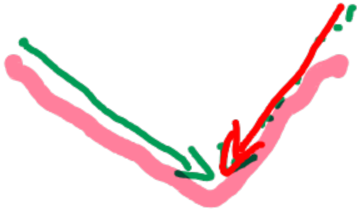
capacity + conservation

induction step: capacity OK by choice of δ

conservation

Case

forward edge



both sides $+\delta$

one forward one backwards

neither side changes

both backwards

both sides $-\delta$

Properties ~~and running time~~

(2) If c_e integer all edges
⟹ Ford-Fulkerson gives us integer flow
by induction

(3) Each iteration increases flow value by δ

Comming Monday
running time
find the max flow